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SELF-ADJUSTMENT EFFECTS IN A LASER DRIVEN BURNING

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The quasi-equilibrium dense discharge plasma at atmospheric pressure and temperature $T \sim 1$ eV which is sustained by the radiation from a neodymium glass laser is the object of intensive investigations and finds numerous applications in engineering [1, 2]. The laser driven burning was obtained by Bunkin et al. [3] for the first time. The subsequent investigations have shown that the discharge is of a threshold character related to the radiation intensity I_c ($I_c \approx 10$ MW/cm^2). For $I > I_c$, the discharge front propagates along the light channel at a velocity of the order of tens of meters per second toward both sides of the focus. The discharge plasma is optically transparent (the absorption coefficient is $\mu \sim 10^{-2}$ cm^{-1}), its parameters (temperature and density) are, on average, constant in time and uniform in space within the light channel. The mean electron density in the plasma is $n_e \approx 2 \cdot 10^{17}$ cm^{-3}, and the pressure is equalized in space owing to a subsonic propagation regime. The discharge wavefront velocity depends on the intensity of an external source and increases according to the law $V_f \propto \sqrt{I}$ as the threshold intensity I_c is exceeded severalfold. In the threshold region, one can observe interesting effects associated with fluctuations of the front velocity of the order of $\Delta V_f \sim 1$–2 m/sec. In this case, the front profile does not vary in shape until the complete stoppage. The temperature and density measurements performed by Bukatyi et al. [4] also indicate the complicated character of the motions in the discharge plasma, which is reflected in macroscopic fluctuations of the discharge parameters.

The fist theoretical discharge model was proposed by Yu. P. Raizer [1]. In this model, he used the similarity between the burning of a Bickford fuse and the discharge motion, as in the subsequent models. He derived the correct dependence of the front velocity on the radiation intensity within this model, which is described by the one-dimensional nonlinear heat-conduction equation.

In the present paper, we proceed from the gas-dynamic equations in which significant nonhydrodynamic energy-transfer mechanisms, namely, heat conduction and radiation, are taken into account in the description of the light-discharge properties. Ignoring the divergence of a light beam and taking into account the optical plasma transparency, we assume that the channel has a cylindrical symmetry, and the radiation flux does not vary with the depth of its penetration into the plasma. In addition, we assume that the major mechanism of energy losses is plasma self-radiation. Note that this is true for a sufficiently large width of the beam d (> 0.1 cm) [1, 2].

1. Governing Equations of the Discharge Model. The description of the discharge properties is based on the following hydrodynamic equations for the fields of density ρ, velocity $\mathbf{V}$, and temperature T:

$$\rho_t + \nabla(\rho \mathbf{V}) = 0, \quad \rho(\mathbf{V}_t + (\mathbf{V}\nabla)\mathbf{V}) = -\nabla p, \quad T_t + \mathbf{V}\nabla T = F + D\Delta T. \tag{1.1}$$

$$F = (\mu I - \Phi)/(c_p \rho), \quad D = æ/(c_p \rho). \tag{1.2}$$

Here Δ is the Laplace operator, p is the pressure, and

$$F = (\mu I - \Phi)/(c_p \rho), \quad D = æ/(c_p \rho), \tag{1.2}$$

where Φ is the energy-flux density of the plasma self-radiation, c_p is the heat capacity under constant pressure, and $æ$ is the heat conductivity. Let the discharge propagate along the z axis. It then follows from (1.1) that

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the steady-state motion of the discharge front obeys the following equations:

$$\rho_0 V_z = 0, \qquad V_f T_z = F + D T_{zz}. \tag{1.3}$$

It is known from the autowave theory [5] that Eqs. (1.3) with the nonlinear function $F(T)$ and the monotonically varied first-order Frechet derivative $\partial F/\partial T$ have solutions in the form of an oscillations-free front:

$$V_f \propto \sqrt{\nu D}, \tag{1.4}$$

where

$$\nu = \frac{\partial F}{\partial T}\bigg|_{T=T_0,\, \rho=\rho_0}; \tag{1.5}$$

$$F(T_0, \rho_0) = \frac{\mu(T_0, \rho_0) I - \Phi(T_0, \rho_0)}{c_p \rho_0} = 0 \tag{1.6}$$

(T_0 and ρ_0 are the unperturbed temperature and density in the discharge).

In studying the stability in the coordinate system related to the discharge, we give solutions in the form

$$T = T_0 + \delta T, \quad \rho = \rho_0 + \delta\rho, \quad \mathbf{V} = \mathbf{e}_z V, \tag{1.7}$$

where δT and $\delta\rho$ are small perturbations: $\delta\rho/\rho_0 \ll 1$ and $\delta T/T_0 \ll 1$. Introducing the smallness parameter $\varepsilon \approx V/c \ll 1$ (c is the velocity of sound), we retain in the equations only the second-order terms relative to ε. Assuming the perturbations to be adiabatic ($\omega \gg k^2 D$, where ω and k are the characteristic frequencies and wave numbers of the perturbations), we write the function k as

$$p = p_0 + c^2 \delta\rho + \frac{1}{2}(\gamma - 1) c^2 \rho_0 \left(\frac{\delta\rho}{\rho_0}\right)^2 + \rho_0 c^2 \beta \delta T \tag{1.8}$$

where γ is the adiabatic exponent and β is the thermal-expansion coefficient. Substituting the solutions in the form (1.7) into (1.1) and taking into account (1.8), we obtain the governing equations of the model in the Boussinesq approximation ($\delta\rho/\rho_0 \sim \varepsilon$ and $\delta T/T_0 \sim \varepsilon^2$)

$$\begin{gathered} \delta\rho_t + \rho_0 V_z = -(\delta\rho V)_z, \\ V_t + \frac{c^2}{\rho_0}\delta\rho_z = -\frac{1}{2}(\gamma - 1) c^2 \left(\frac{\delta\rho}{\rho_0}\right)^2_z - \frac{1}{2}(V^2)_t - c^2 \beta \delta T_z, \\ \delta T_t = \nu \delta T + D \Delta \delta T. \end{gathered} \tag{1.9}$$

Differentiating the first equation with respect to z and the second one with respect to t, we find that

$$(V_t + cV_z)(V_t - cV_z) = -\frac{1}{2}\left[V^2 + (\gamma - 1) c^2 \left(\frac{\delta\rho}{\rho_0}\right)^2\right]_{zt} + \frac{c^2}{\rho_0}(\delta\rho V)_{zz} - c^2\beta(\nu\delta T + D\Delta\delta T)_z. \tag{1.10}$$

We search for the solution of Eqs. (1.10) in the form of quasi-simple waves [6]. One can then make, with due accuracy, the substitution $\partial/\partial t - c\partial/\partial z = -2c\partial/\partial z$ on the left-hand side of (1.10) and set $\partial/\partial t = c\partial/\partial z$, $\delta\rho/\rho_0 = V/c$, and $\delta T/T_0 = (\gamma - 1) V/c$ on the right-hand one. After these transformations and the transition to the reference system moving with velocity c relative to the medium, Eq. (1.10) takes the simpler form

$$V_t = \frac{1}{2}\beta T_0(\gamma - 1)(\nu + D\Delta) - \frac{2-\gamma}{2} V V_z. \tag{1.11}$$

In the above relation, a wave that is traveling in the positive direction along the z axis is described, the nonlinear and dissipative terms are of the same order, and t is "slow" time ($t \to t - z/c$).

2. Stability of a Linearized System. We write Eq. (1.11) as follows:

$$V_t = L(\lambda) V + h(V, \lambda), \tag{2.1}$$

where $L = (1/2)(\gamma - 1)\beta T_0(\nu + D\Delta)$ is the linear operator that acts in space and in terms of which the function $V(\mathbf{r}, t)$ is defined, the term $h(V, \lambda)$ incorporates the nonlinear character of the right-hand side of Eq. (1.11), and λ reflects the dependence of the solution on the problem parameters (ν and D).

We first search for solutions of the linear auxiliary problem

$$V_t = L(\lambda)V. \tag{2.2}$$

Since system (2.1) is autonomous, Eq. (2.2) allows solutions of the form

$$V = u(\mathbf{r}) \exp(\lambda t). \tag{2.3}$$

Substituting (2.3) into (2.2) and using the explicit form of the operator L in the cylindrical coordinate system, we have

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}\right)u = -K^2 u \qquad \left[K^2 = \frac{1}{D}\left(\nu - \frac{2\lambda}{(\gamma-1)\beta T_0}\right)\right]. \tag{2.4}$$

Supplemented by the appropriate boundary conditions, this equation defines the eigenvalue problem. For example, for the case of axial symmetry with the boundary condition $u(r) = 0$ for $r = r_0$ (r_0 is the radius of the light beam), it is easy to find from (2.4) the following eigenfunctions and eigenvalues:

$$u = J_0(k_\perp r)\exp(ik_z z), \qquad k_\perp \approx 2.40/r_0, \qquad \lambda = (1/2)(\gamma-1)\beta T_0(\nu - (k_\perp^2 + k_z^2)D). \tag{2.5}$$

Here $J_0(k_\perp r)$ is the Bessel function and k_z is the wave number.

Thus, the solution of the linearized problem (2.2) is

$$V(\mathbf{r}, t) = au(\mathbf{r})\exp(\lambda t), \tag{2.6}$$

where u and λ are determined by relations (2.5), and a is a constant. One can easily notice that the character of the solution depends on the eigenvalue of the parameter λ (or, what is the same, on the governing parameter ν). For $\nu \geqslant k^2 D$, instability arises in the system; the critical point

$$\nu_c = (k_\perp^2 + k_z^2)_c D, \qquad \lambda_c = 0 \tag{2.7}$$

corresponds to a regime that is intermediate between the asymptotic stability and instability of the system and determines the threshold condition for the existence of the discharge. It is important to note that owing to (2.5) and (2.7), the unstable mode k_c is completely determined by the system parameters and characterizes the spatial perturbation length of a steady-state solution. Thus, we have the mechanism of intrinsic-wavelength generation in an initially uniform system. One can expect that, for $\lambda > \lambda_c$, this perturbation will determine the basic properties of the system [7].

3. Evolution of a Nonlinear System. We return to an analysis of the dynamics of the system described by the nonlinear equation (2.1). We confine ourselves to the case of bifurcation of the solution near the critical point λ_c (2.7). Since the condition

$$\frac{d}{d\nu}\lambda(\nu)\Big|_{(\lambda=\lambda_c)} = (1/2)(\gamma-1)\beta T_0 \neq 0$$

is satisfied, in our case it follows from the general results of the self-adjustment theory [7] that the solutions that arise for $\lambda \geqslant \lambda_c$ are stable and steady.

This means that we need to find the solution of the equation

$$L(\lambda)V + h(V, \lambda) = 0, \qquad h(V) \equiv -\frac{2-\gamma}{2}VV_z. \tag{3.1}$$

With allowance for the character of the nonlinearity, we find the solution of (3.1) as a series in the smallness parameter ε:

$$V = \varepsilon V_1 + \varepsilon^2 V_2 \exp(ik_z z) + \ldots + \text{c.c.}, \tag{3.2}$$

in which, according to (2.6), we should set

$$V_1 = a(t)J_0(k_\perp r)\exp(ik_z z), \tag{3.3}$$

where $a(t)$ is a desired time-dependent function. Substitution of the series (3.2) together with (3.3) offers the possibility of defining V_2:

$$V_2 = -(2(2-\gamma)/3(\gamma-1)\beta T_0)a^2(t)J_0^2(k_\perp r)ik_z\exp(2ik_z z). \tag{3.4}$$

Using the expansion (3.2) together with (3.3) and (3.4) in Eq. (2.1), we find the equation that defines the function $a(t,R)$:

$$a_t + \lambda_0(R-1)a - \sigma a^3 = 0. \tag{3.5}$$

Here

$$\lambda_0 = (1/2)(\gamma-1)\beta T_0\nu_c,\quad R=\nu/\nu_c,\quad \sigma=(2-\gamma)^2\langle J_0^2\rangle/3D(\gamma-1)\beta T_0,$$
$$\langle J_0^2\rangle = \frac{1}{r_0^2}\int_0^{r_0} J_0^2(k_\perp r)r\,dr = \frac{1}{2}J_1^2(k_\perp r_0). \tag{3.6}$$

Equation (3.5) with parameters (3.4) has the well-known solution and describes the supercritical bifurcation of the system. For $R > 1$, this equation admits two solutions

$$a_s(R) = \pm\sqrt{\frac{\lambda_0}{\sigma}(R-1)}, \tag{3.7}$$

which are asymptotically stable and are realized in the characteristic time $\tau \approx (1/2)\lambda_0(R-1)$. The mathematical reflection of the system's qualitative behavior, which is due to the bifurcation at the point $R_c = 1$, is the singularity leading to the non-analytical character of the solution in the vicinity of the critical point.

The above analysis of the dynamics of the nonlinear system (2.1) shows that the evolution of natural perturbations leads to the formation of coherent dissipative structures (DS) in the system, which are described by the formula $V(\mathbf{r},t) = a(t)J_0(k_\perp r)\exp(ik_z z)$, where $a(t)$ is the function subject to Eq. (3.5), and the eigenvalues of the parameters are determined by relations (2.5) and (2.7).

4. Effect of Random Sources. In a real experiment, the laser radiation can fluctuate, which, in principle, affects the dynamics of the problem. In view of this, we consider the effect of random forces without indicating their source.

Note that Eq. (3.5) has a formal similarity with the corresponding equations describing the second-order nonequilibrium phase transitions [8]. In fact, the singularity at the critical point $R_c = 1$, which is connected with a transition of the system from one to another state, requires taking into account the effect of fluctuation forces.

Assume that a Lagrangian source with power $Q\delta(t-t') = \langle F(t)F(t')\rangle$, where $\delta(t-t')$ is the delta function, acts in system (3.5). The evolution equation of the system then takes the form

$$a_t = \lambda_0(R-1)a - \sigma a^3 + F(t), \tag{4.1}$$

where $F(t)$ is a random force.

We introduce the function $f(a,t)$ in the "coordinate" space $a(R)$. The Fokker–Plank equation, which describes the time variation in the distribution function $f(a,t)$ and the corresponding (4.1), takes the form

$$\frac{\partial f(a,t)}{\partial t} = Q\frac{\partial^2 f}{\partial a^2} + \frac{\partial}{\partial a}[(\lambda_0(R-1)a - \sigma a^3)af],\quad \int f(a)\,da = 1. \tag{4.2}$$

In a steady state, the solution of this equation is

$$f(a) = C\exp[-(L/2Q)(\sigma a^4/2 - \lambda_0(R-1)a^2)]. \tag{4.3}$$

For the distribution (4.3), there are two most probable values

$$a_s = \pm\sqrt{\frac{\lambda_0}{\sigma}(R-1)}, \tag{4.4}$$

which coincide for $R_c = 1$.

Let us calculate the mean square $\langle a^2\rangle = \int a^2 f(a)\,da$. Using (4.3), we find

$$\langle a^2\rangle = \frac{\lambda_0}{\sigma}(R-1), \qquad R > 1, \tag{4.5}$$

$$\langle a^2\rangle = \sqrt{\frac{4Q}{\sigma}}\,\frac{\Gamma(3/4)}{\Gamma(1/4)}, \qquad R = 1, \tag{4.6}$$

where $\Gamma(3/4)$ and $\Gamma(1/4)$ are the gamma functions. Clearly, for $R > 1$ (far enough from R_c), the squares of relations (3.7) and (4.4) coincide with (4.5); for $R = R_c$, relation (4.6) characterizes in our case the r.m.s. fluctuations of the a_s-order parameter.

We evaluate fluctuations in the critical point assuming that the source of random forces are fluctuations of the laser radiation $\delta I(t)$ [8]. Here the term $(\mu I/\rho_0 c_p)$ $(\delta I/I)$ appears in the third equation in (1.1). Performing transformations which are similar to those used in the derivation of Eq. (1.11), we find

$$Q\delta(t-t') = \left(\frac{c\mu I}{\rho_0 c_p T_0}\right)^2 \frac{1}{4\langle J_0^2\rangle}\,\frac{\langle \delta I(t)\delta I(t')\rangle}{I_0^2}\,\delta(t-t').$$

Having substituted this relation and the σ value from (3.6) into formula (4.6), we estimate, in order of magnitude, the level of fluctuations at the critical point $R_c = 1$:

$$\langle a^2\rangle \sim (c\mu I/[(2-\gamma)\rho_0 c_p T_0])\sqrt{(\gamma-1)\beta T_0 D t_c(\langle \delta I^2\rangle/I^2)}. \tag{4.7}$$

Here t_c is the typical time of laser-radiation fluctuation correlation, and $\langle \delta I^2\rangle/I^2$ is their relative level.

We dwell upon the physical meaning of the relations derived above. The field function $V(\mathbf{r}, t)$ describes the particle velocity in the plasma. In a laboratory coordinate system, this function is of the form of a traveling wave

$$V = a_s(R) J_0(k_\perp r)\exp\left[-i\omega t + i k_z(z - z_f)\right], \qquad R > 1,$$

where $\omega = k_z c$ is the sound-wave frequency and $z_f = V_f t$ is the front coordinate. With the use of the boundary condition $\partial V/\partial z\big|_{z=z_f} = 0$, this expression takes the form

$$V(r,z) = V_f(R) J_0(k_\perp r)\cos\left[k_z(z-z_f)\right]\exp(-i\omega t); \tag{4.8}$$

$$V_f(R) = \left(\sqrt{3/2}(\gamma-1)\beta T_0/(2-\gamma)\right)k_\perp D\sqrt{R-1}, \qquad R > 1. \tag{4.9}$$

In deriving (4.9), we have taken into account conditions (3.6) and (3.7) and performed the corresponding averagings over time and over the cross section of the plasma channel. Note that it follows from (3.6) and (3.7) that the dependence of the front velocity on the degree of supercriticality $V_f \propto \sqrt{R-1}$, and the velocity profile, which is described by the function $J_0(k_\perp r)$, does not vary its shape (as noted by Bufetov et al. [2]). For $R = 1$, formulas (4.8) and (4.9) lose their value. In this state, the dynamics of the system is determined by the power of random sources, and relation (4.7) describes, in essence, random velocity oscillations near the zero value (this effect was experimentally supported in [2]). Note also that relation (4.4) reflects the equal probability of the appearance of two fronts (leading and trailing) of discharge motion.

We perform a quantitative estimation of the quantities for typical values of the problem parameters [1, 2]: plasma density $\rho_0 \approx 2\cdot 10^{-4}$ g/cm^3 under normal pressure, equilibrium electron temperature $T_0 = T_e \approx 1.3$ eV, equilibrium electron density $n_e \approx 2\cdot 10^{17}$ cm^{-3}, $\mu \approx 10^{-2}$ cm^{-1}, $r_0 \approx 0.5$ cm, $B \approx 0.5$, $\gamma \approx 1.25$, $\beta T_0 \approx 1$, $D \approx 2\cdot 10^3$ cm^2/sec, and $c \approx 2\cdot 10^5$ cm/sec. We start with the determination of the threshold energy characteristics of the pumping field. It follows from (2.7) and definitions (1.5) and (1.6) that the minimum value of the field $B(\mu I)_c/\rho_0 c_p T_0 = k_\perp^2 D$ is reached if the condition $(k_z/k_\perp)^2 \ll 1$ is satisfied. Substituting the typical values of the problem into this relation, we find the threshold radiation power $P_c = \pi r_0^2 I_c = (2.4)^2 \pi D \rho_0 c_p T_0 / B\mu \approx 1$ MW. Under the same condition, with allowance for the wave adiabaticity ($\omega > k_\perp^2 D$), we calculate, from (4.8) and (4.9), the discharge front velocity $V_f = 20\sqrt{R-1}$ m/sec, the level of density fluctuations of the particle number in the discharge plasma $\delta n/n_0 \approx 10^{-2}$, and the wave frequency $\nu = \omega/2\pi \sim 10$ kHz (fluctuations of the plasma parameters and discharge-induced sound oscillations with a given frequency were observed by Bukatyi et al. [4, 9] for the first time). We determine the behavior of the discharge front at the critical point $R = 1$ using (4.7). For $\sqrt{\langle \delta I \rangle^2 / I^2} \sim 10^{-5}$ and $t_c \sim 10^{-5}$ sec, we find the level of discharge-front velocity fluctuations in the threshold domain $\sqrt{\langle \Delta V_f^2 \rangle} = \sqrt{\langle a^2 \rangle} \sim 1$ m/sec.

5. Discussion of Results. Conclusions. We have derived, in the Boussinesq approximation, an equation that describes the evolution of the laser driven burning in a Nd laser-generated field. We have shown that at a definite (threshold) value of the external field, the hydrodynamic-type instability appears in the system [10], the role of the Rayleigh number being played by the relation $R = \nu(I)/k_c^2 D$, where $\nu(I)$ is the characteristic frequency of the energy contribution to the discharge, which is parameter-dependent on the external field, D is the heat-diffusion coefficient, and k_c is the wave number determined by the characteristic parameters of the system (the effects associated with viscosity η have been ignored, because $\eta/D \ll 1$ in the discharge plasma [1]). The dynamics of the forming nonlinear sound wave with an axial symmetry and a slowly varying amplitude has been studied. It has been shown that the evolution of the envelope obeys the Ginzburg–Landau equation. The effect of random sources on the given system has been studied. The fluctuations that develop in the system have been shown to determine the threshold value of the pumping field and to show up as the macroscopic (directed) motion of the discharge.

The region of applicability of the model considered is limited by the physical condition of optical transparency, i.e., the condition $\omega_l/\omega_t \ll 1$, where ω_l is the Langmuir frequency and ω_t is the electromagnetic-wave frequency (Nd laser), should be satisfied. Another limitation is connected with the level of laser radiation fluctuations $\delta I/I \sim (V/c)^2 \sim 10^{-4}$. As a rule, these conditions are satisfied.

Realizations of the mathematical models in a physical situation (threshold power, velocity and profile of the front, level and frequency of fluctuations, front behavior in the critical region, and fluctuations of the discharge wavefront velocity ΔV_f) are in qualitative and quantitative agreement with experimental data.

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